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Interaction of water waves with two closely spaced vertical obstacles

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Two-dimensional waves are incident upon a pair of vertical flat plates intersecting the free surface in a fluid of infinite depth. An asymptotic theory is developed for the resulting wave reflexion and transmission, assuming that the separation between the plates is small. The fluid motion between the plates is a uniform vertical oscillation, matched to the outer wave field by a local flow at the opening beneath the plates. It is shown that the reflexion and transmission coefficients undergo rapid changes, ranging from complete reflexion to complete transmission, in the vicinity of a critical wavenumber where the fluid column between the obstacles is resonant.

1. Introduction

When two long cylindrical obstacles are placed in parallel on the free surface, in the presence of normally incident plane progressive waves, interference effects will persist between the two obstacles even when their spacing is large compared with the wavelength. Thus, in general, there will exist an infinite set of wavelengths λ, or wavenumbers $K = 2\pi/\lambda$, for which the obstacle pair is 'transparent', i.e. there is complete transmission and no reflexion of the incident wave system.

The possibility of complete reflexion is less obvious and, intuitively, this might be regarded as unlikely or impossible. However, Evans & Morris (1972) have demonstrated the occurrence of complete reflexion for a pair of vertical barriers, which extend down from the free surface to a depth a, separated by a distance $2b$. With the usual assumptions of linearized two-dimensional motion in a fluid of infinite depth, this problem has an exact but complicated solution, due to Levine & Rodemich (1958). Evans & Morris (1972) derive an approximate but more descriptive solution, based on complementary variational approximations, and prove the existence of complete reflexion and transmission for two infinite sets of critical wavenumbers. Bounds are obtained on the critical wavenumbers for the interesting case of complete reflexion, but the variational procedure of Evans & Morris gives close bounds only for large barrier spacing. In this case the occurrence of complete reflexion is effectively masked by the fact

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that the critical wavenumbers are large, and hence the transmission coefficient is exponentially small in Ka. Evans & Morris (1972) note that the lowest critical wavenumber is reduced for smaller values of the barrier spacing ratio b/a, but that in this case the variational approximation is "poor", and "it is only possible to obtain bounds for the smallest root... Thus it is found that (for $b/a = 0{\cdot}1$) $0{\cdot}89 < Ka < 0{\cdot}99$ and (for $b/a = 0{\cdot}05$) $0{\cdot}87 < Ka < 1{\cdot}03$."

In spite of the 'poor' approximation obtained by Evans & Morris (1972) for small values of b/a, the proximity of the critical values of Ka to $1{\cdot}0$ should be noted. If the spacing ratio b/a is sufficiently small, the water column between the barriers will oscillate in a simple vertical mode, in the same manner as a floating rigid body of width $2b$ and depth a. For such a body the hydrodynamic added-mass and damping forces will be negligible if the 'slenderness' ratio b/a is sufficiently small. A simple calculation equating the inertial force, due to vertical acceleration of the displaced fluid mass, to the hydrostatic buoyancy force (or, equivalently, the kinetic and potential energies) then shows that resonance will occur at a frequency $2\pi\omega$ such that $\omega^2 a = g$, where g is the acceleration due to gravity (cf. Newman 1963). For deep-water incident waves, where $K = \omega^2/g$, the equivalent condition is $Ka = 1$, and in the vicinity of this resonance the fluid column may influence the exterior wave field out of proportion to the small mass of fluid between the obstacles.

Here we shall analyse the same problem as Evans & Morris (1972), but using a singular-perturbation method based on the assumption that $b/a \ll 1$. In this way it will be possible to obtain explicit results for the reflexion and transmission coefficients and for the vertical motion of the fluid column between the two obstacles. Our approach is based on a rather loose application of the method of matched asymptotic expansions, following a viewpoint similar to that used by Tuck (1971) to analyse wave transmission through a small gap in a single infinite vertical barrier. Three separate flow regimes are considered initially, including (i) the thin vertical column of fluid between the obstacles, (ii) the local flow near the lower edges of the obstacles and (iii) the outer solution comprising the wave field exterior to the obstacles. The thin fluid column is in a simple state of uniform vertical oscillatory flow, with the free-surface boundary condition imposed in a relatively trivial fashion. At the entrance, near the lower edges of the obstacles, the local solution is that of a two-dimensional potential flow leaving the opening between two semi-infinite flat plates and ejecting into an unbounded outer field in an oscillatory source-like manner. Finally, in the outer field, the two obstacles are effectively collapsed into a single plane barrier, and the solution is a superposition of the well-known single-barrier solution and an oscillatory wave source at the lower edge, this source representing the flow into and out of the entrance in the inner region.

In §2 the boundary-value problem is formulated, and in §3 the outer solution is developed in terms of the known single-barrier solution of Ursell (1947) and others. In §4 the vertical column and local flow near the entrance are treated as a single composite inner flow and matched with the outer solution. In §5 the reflexion and transmission coefficients are computed, and the occurrence of both complete reflexion and complete transmission is noted. The response of the

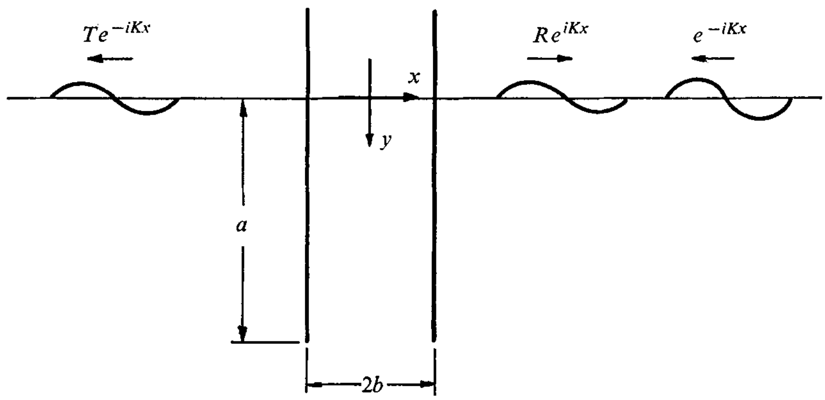

FIGURE 1. Co-ordinate system and barrier configuration including the transmitted, reflected and incident wave systems.

fluid column is analysed in § 6, and a highly tuned resonant peak is shown to occur. The results are discussed from a practical standpoint in § 7.

2. Formulation of the problem

Following Evans & Morris (1972), Cartesian co-ordinates (x, y) are employed with the origin in the undisturbed free surface and y vertically downwards. Plane progressive waves of frequency $\omega/2\pi$ are incident from $x = +\infty$ upon two vertical barriers which occupy $x = \pm b$, $0 < y < a$. This configuration is shown in figure 1. With the usual assumptions of two-dimensional linearized inviscid flow, the fluid velocity is equal to the gradient of a velocity potential $\mathrm{Re}\{\phi(x,y)\,e^{-i\omega t}\}$, with $\phi(x,y)$ governed by the boundary-value problem

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = 0, \quad y > 0, \tag{2.1}$$

$$K\phi + \partial\phi/\partial y = 0, \quad y = 0, \quad K = \omega^2/g, \tag{2.2}$$

$$\partial\phi/\partial x = 0, \quad x = \pm b, \quad 0 < y < a. \tag{2.3}$$

The boundary conditions (2.2) and (2.3) are supplemented by the requirements that $\phi \to 0$ as $y \to \infty$ and $\nabla\phi$ should have, at most, a square-root infinity at the sharp edges $x = \pm b$, $y = a$. Finally, radiation conditions are imposed in the form

$$\phi(x,y) \simeq e^{-Ky}\,(e^{-iKx} + R\,e^{iKx}), \quad x \to +\infty, \tag{2.4}$$

and

$$\phi(x,y) \simeq T\,e^{-Ky-iKx}, \quad x \to -\infty. \tag{2.5}$$

Here R and T are the complex reflexion and transmission coefficients, and the incident wave potential is assumed to be of unit amplitude.

A solution is sought under the assumption that $b/a = \epsilon$ is a small parameter, with $Ka = O(1)$. Thus the two barriers are closely spaced, relative to the depth a and wavelength $\lambda = 2\pi/K$. Using the method of matched asymptotic expansions and an approach similar to that of Tuck (1971), we consider separately an outer region $(Kx, Ky) = O(1)$ and an inner region $x/b = O(1)$, $0 < y \gtrsim a$. (Strictly speaking, the inner region might include only the vicinity of the opening between the barriers $x/b = O(1)$, $(y-a)/b = O(1)$, and the remaining column of fluid above this between the barriers would be regarded as a second outer region, to be coupled to the first by the flow through the inner region. But in view of the relatively trivial role of the free surface between the barriers, this formulation is not necessary here, it being simpler to regard the entire fluid column $|x| < b$, $y < a$ together with the opening at $y = a$ as a 'composite' inner region.)

3. The outer solution

In the outer region the double barrier is effectively collapsed, as $\epsilon \to 0$, into a single barrier $x = 0$, $0 < y < a$, together with a source-like flow at the opening $x = 0$, $y = a$ which accounts for the mass flux m into the inner region. Thus the outer solution will take the form

$$\phi(x, y) = \phi_0(x, y) + mG(x, y, a), \tag{3.1}$$

where $\phi_0(x, y)$ is the potential for a single vertical barrier ($\epsilon = 0$), G is the Green's function or source potential and m is the (unknown) source strength. The solution ϕ_0 for a single barrier is well known, cf. Ursell (1947) or Wehausen & Laitone (1960); we shall not require the full solution, but only the reflexion and transmission coefficients for the single barrier, which are given by

$$R_0 = \pi I_1(Ka)/[\pi I_1(Ka) + iK_1(Ka)], \tag{3.2}$$

$$T_0 = iK_1(Ka)/[\pi I_1(Ka) + iK_1(Ka)]. \tag{3.3}$$

Here I_1 and K_1 are the usual modified Bessel functions, of order one. In addition we shall need the inner limit $\phi_0(0, a)$, which can be found by first noting that

$$\phi_0 = e^{-Ky-iKx} + \phi_d, \tag{3.4}$$

where ϕ_d is the diffraction potential due to the presence of the single barrier. From symmetry the potential ϕ_d is an odd function of x, continuous on $x = 0$, $y > a$, with a square-root zero at the edge $y = a$. Thus ϕ_d vanishes on $x = 0$ for $y > a$, and the inner limit of ϕ_0 is simply

$$\phi_0(0, a) = e^{-Ka}. \tag{3.5}$$

The Green's function G appearing in (3.1) is also well known, cf. Wehausen & Laitone (1960), and can be expressed in the form

$$G(x, y, a) = \frac{1}{4\pi}\log\left[\frac{x^2+(y-a)^2}{x^2+(y+a)^2}\right] - \frac{1}{\pi}\fint_0^\infty \frac{e^{-k(y+a)}\cos kx}{k-K}\,dk - i\,e^{-K(y+a)}\cos Kx. \tag{3.6}$$

Here $\fint$ denotes the principal-value integral.

It follows, from (3.5) and (3.6), that the inner limit of the outer solution (3.1), valid for $r = [x^2+(y-a)^2]^{\frac{1}{2}} \to 0$, is given by

$$\phi(x, y) = (m/2\pi)\log(r/2a) + e^{-Ka} + (m/\pi)\,e^{-2Ka}\,\mathrm{Ei}\,(2Ka) - im\,e^{-2Ka} + O((r/a)^{\frac{1}{2}}), \quad r/a \ll 1, \tag{3.7}$$

where Ei is the exponential integral

$$\mathrm{Ei}(2Ka) = -\int_{-2Ka}^{\infty} \frac{e^{-u}}{u}\, du. \tag{3.8}$$

The outer solution is now complete, except for the source strength m, which must be determined by matching with the inner solution.

4. The inner solution and matching

In the inner region, between the two obstacles and in the vicinity of their lower edges, the fluid motion is simply an oscillatory flow between two semi-infinite plates $x = \pm b$, $-\infty < y < a$. Using the complex potential $\phi + i\psi$, this flow is described by the implicit solution (cf. Lamb 1932, §66)

$$x + i(y-a) = (ib/\pi)\{\exp[-\alpha(\phi + i\psi) + \beta] - \alpha(\phi + i\psi) + 1 + \beta\}. \tag{4.1}$$

Here the streamlines $\psi = \pm\pi/\alpha$ coincide with the obstacles $x = \pm b$, and α and β are arbitrary real constants. The asymptotic approximation to (4.1) far above the lower edges $y = a$ in the inner region is

$$x + i(y-a) \simeq (ib/\pi)[-\alpha(\phi + i\psi) + 1 + \beta], \quad \alpha\phi - \beta \gg 1. \tag{4.2}$$

Thus the streaming flow in the column is governed by the potential

$$\phi \simeq -\pi y/b\alpha + (1 + \pi a/b + \beta)/\alpha, \quad (y-a)/b \ll 1, \tag{4.3}$$

and the constant β can be determined by the free-surface boundary condition (2.2) as

$$\beta = \pi/Kb - \pi a/b - 1. \tag{4.4}$$

The remaining constant α must be determined by matching the outer limit of the inner solution (4.1) to the inner limit of the outer solution. For this purpose we first obtain the outer approximation of (4.1) in the implicit form

$$x + i(y-a) \simeq (ib/\pi)\exp[-\alpha(\phi + i\psi) + \beta] \tag{4.5}$$

or, with $|x + i(y-a)| = r$,

$$\phi \simeq (-1/\alpha)\log(\pi r/b) + \beta/\alpha + O((b/r)\log^2(r/b)), \quad r/b \gg 1. \tag{4.6}$$

On matching of (3.7) and (4.6) in the overlap region $b \ll r \ll a$, it follows that

$$m = -2\pi/\alpha \tag{4.7}$$

and hence, matching the $O(1)$ terms to eliminate α and using (4.4), the source strength m is given by

$$m = 2\pi Kb\, e^{-Ka}\{\pi(Ka-1) + Kb\log(2\pi a/b) + Kb[1 - 2e^{-2Ka}\mathrm{Ei}(2Ka) + 2\pi i e^{-2Ka}]\}^{-1}. \tag{4.8}$$

This completes the solution of the problem, the source strength m in the outer solution (3.1) and the constants α and β in the inner solution (4.1) being determined by (4.4), (4.7) and (4.8).

b/a	$Ka\ (R = 0)$	$Ka\ (T = 0)$
0	1·0	1·0
0·01	0·980	0·989
0·05	0·929	0·969
0·1	0·885	0·961
0·2	0·822	0·963

TABLE 1. Values of Ka for zero reflexion ($R = 0$) and zero transmission ($T = 0$)

5. The reflexion and transmission coefficients

The reflexion coefficient R and transmission coefficient T in the radiation conditions (2.4) and (2.5) can be computed from the far-field approximations of the outer solution (3.1). The limiting form of the Green's function (3.6) is readily found by contour integration as

$$G(x,y,a) \simeq -i\exp[-K(y+a)+iK|x|], \quad |x| \to \infty. \tag{5.1}$$

Thus, from (2.4) and (2.5),

$$\begin{pmatrix} R \\ T \end{pmatrix} = \begin{pmatrix} R_0 \\ T_0 \end{pmatrix} - ime^{-Ka}, \tag{5.2}$$

where R_0 and T_0 are the single-barrier coefficients given by (3.2) and (3.3). Using (4.8) to eliminate m gives

$$\begin{pmatrix} R \\ T \end{pmatrix} = \begin{pmatrix} R_0 \\ T_0 \end{pmatrix} - 2\pi iKb\,e^{-2Ka}\{\pi(Ka-1) + Kb\log(2\pi a/b) + Kb[1-2e^{-2Ka}\mathrm{Ei}(2Ka)+2\pi i\,e^{-2Ka}]\}^{-1}. \tag{5.3}$$

Equation (5.3) in conjunction with (3.2) and (3.3) is the desired relation for the reflexion and transmission coefficients. It is not difficult to confirm that $|R|^2+|T|^2 = 1$, and hence energy is conserved. In general R and T differ from R_0 and T_0 by $O(Kb)$, and thus the small gap between the two obstacles exerts a correspondingly small influence on the outer flow. However, this influence is magnified when $Ka \simeq 1$, a situation corresponding to resonant oscillations of the fluid column between the obstacles. It is clear from (5.3) that, when

$$Ka = 1-(Kb/\pi)\log(2\pi a/b)+O(Kb), \tag{5.4}$$

the effect of the gap is in fact $O(1)$. Indeed, in these circumstances it is possible to have either complete transmission or complete reflexion. The approximate relations for these conditions are readily calculated by setting $Ka = 1$ in (5.3), except in the first term in braces, and thus

$$\frac{Ka-1}{Kb} = -\frac{1}{\pi}\log\frac{2\pi a}{b}+\begin{pmatrix} 0{\cdot}0168 \\ 0{\cdot}907 \end{pmatrix}+O\left(Kb\log\frac{2\pi a}{b}\right) \quad \text{for} \quad \begin{pmatrix} R=0 \\ T=0 \end{pmatrix}. \tag{5.5}$$

The corresponding values of Ka are listed in table 1. It may be noted that the critical wavenumbers for $T = 0$ are consistent with the upper and lower bounds ($0{\cdot}87 < Ka < 1{\cdot}03$ for $b/a = 0{\cdot}05$, and $0{\cdot}89 < Ka < 0{\cdot}99$ for $b/a = 0{\cdot}1$) given by Evans & Morris (1972).

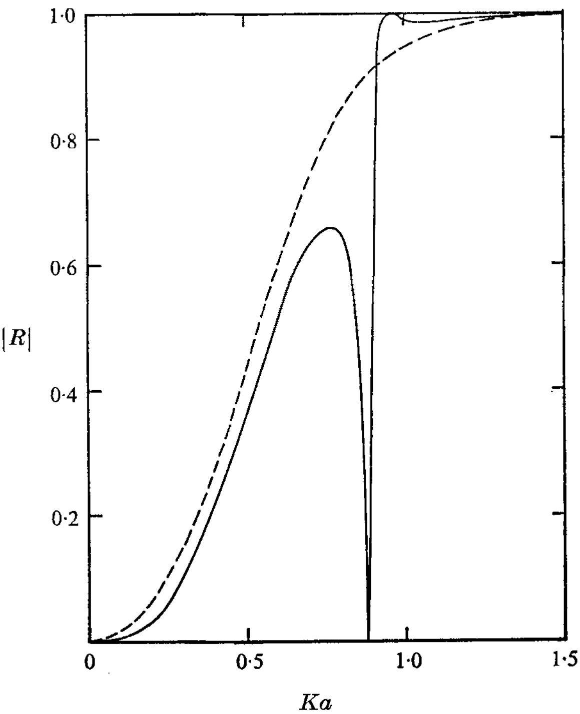

FIGURE 2. Reflexion coefficient of a pair of vertical barriers with spacing ratio $b/a = 0{\cdot}1$. – – –, reflexion coefficient $|R_0|$ for a single barrier.

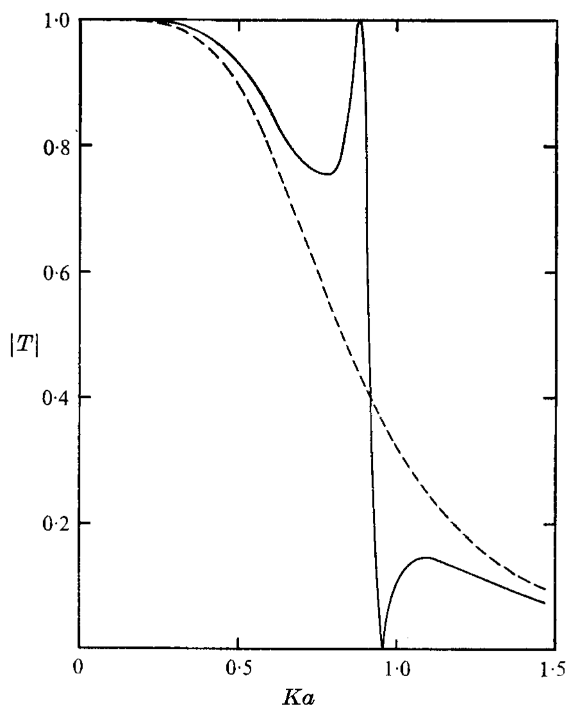

FIGURE 3. Transmission coefficient of a pair of vertical barriers with spacing ratio $b/a = 0{\cdot}1$. – – –, transmission coefficient $|T_0|$ for a single barrier.

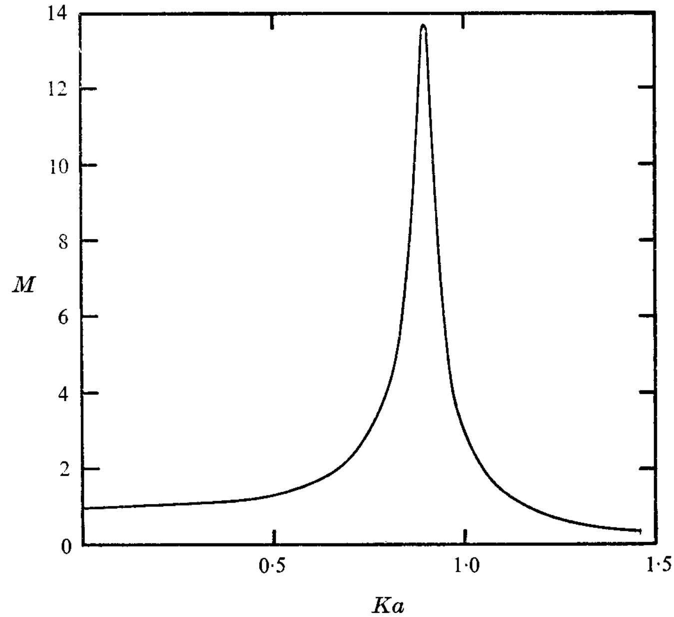

FIGURE 4. Magnification factor of the response between the two vertical barriers with spacing ratio $b/a = 0{\cdot}1$.

Computations of the magnitude of the reflexion and transmission coefficients, based on (5.3), are shown in figures 2 and 3 for the case $b/a = 0{\cdot}1$. Also shown for comparison are the corresponding values of R_0 and T_0, for the single barrier. These figures confirm the occurrence of zero reflexion and complete transmission at $Ka \doteq 0{\cdot}88$, and the converse case of zero transmission and complete reflexion at $Ka \doteq 0{\cdot}96$. For long wavelengths or small barrier depths ($Ka < 0{\cdot}92$) the transmission coefficient is increased relative to the single-barrier values, whereas for short wavelengths or deep barriers ($Ka > 0{\cdot}92$) the transmission coefficient is reduced by the presence of the second barrier.

6. Response between the obstacles

In view of the assumption that $Kb \ll 1$, the free-surface elevation between the two obstacles can be regarded as spatially uniform, and we shall define the 'magnification factor' M as the ratio of the magnitude of this elevation to the incident wave amplitude. Using (4.3), (4.4) and (4.7) it follows that

$$\begin{aligned} M &= |\phi(0,0)| = |m|/2Kb \\ &= \pi e^{-Ka}|\pi(Ka-1) + Kb[\log(2\pi a/b) + 1 - 2e^{-2Ka}\mathrm{Ei}(2Ka) + 2\pi i e^{-2Ka}]|^{-1}. \end{aligned} \tag{6.1}$$

For $b \to 0$ the response is unbounded at $Ka = 1$, but for $b > 0$ radiation damping results in a bounded resonance. Nevertheless this resonance is highly tuned, as shown by the calculated values displayed in figure 4, for the case $b/a = 0{\cdot}1$. In this case the maximum value $M = 13{\cdot}7$ occurs at $Ka = 0{\cdot}894$, or nearly coincident with the wavenumber for zero reflexion. At the zero-reflexion wavenumber,

$M \doteq 12{\cdot}5$, whereas, at the zero-transmission wavenumber, $M \doteq 4{\cdot}7$. Presumably this difference can be explained by noting that at these wavenumbers the single-barrier reflexion is nearly complete ($R_0 \doteq 0{\cdot}9$) and a relatively small perturbation at the lower edge of the barriers is sufficient to block transmission completely. On the other hand, a much larger perturbation is required to cancel the reflected wave and effect complete transmission.

In the two limiting cases, $M \to 1$ for $Ka \to 0$ and $M \to 0$ for $Ka \to \infty$. Thus, for long wavelengths, the response between the obstacles is identical to the incident wave amplitude, whereas for very short waves where there is complete reflexion from the first obstacle, the motion between the two obstacles is negligible.

7. Discussion

The results shown in figures 2–4 for the reflexion and transmission coefficients and the magnification factor of the response between the two obstacles confirm the resonant response near $Ka = 1$ which was anticipated in § 1, and the associated variations of the reflexion and transmission coefficients. The occurrence of complete reflexion is consistent with the results of Evans & Morris (1972), but the practical significance of this result is offset by the occurrence of complete transmission at an adjacent wavenumber. These two critical wavenumbers differ by only 10 %, corresponding to a 5 % difference in frequency. Thus it is unlikely that the characteristics of complete reflexion or transmission can be exploited in practice, in a realistic wave spectrum.

For larger values of the barrier spacing $2b$, the bandwidth of the peaks in figures 2–4 may increase, but the approach used here would be invalid, owing to the finite separation distance. In view of the fact that Evans & Morris's (1972) analysis based on complementary variational formulations gives useful computations for $b/a \geqslant 1$, whereas the present approach is limited to values of $b/a \ll 1$, it would be desirable to perform numerical computations based on the exact theory of Levine & Rodemich, in the relatively narrow range bridging the gap between these two regimes. This would complete our understanding of the very interesting interactions which can occur between two parallel obstacles.

The above theories are also restricted, of course, by the usual assumptions of ideal flow and linearized free-surface effects. Viscous separation may be anticipated near the lower edges of the obstacles, unless these are rounded off, and the linearization assumption may be seriously inaccurate near the resonant wavenumber, at which the response between the barriers is greatly magnified. Nevertheless, the present results may be qualitatively indicative of the pronounced interference which can occur between pairs of long parallel obstacles, and of the possibilities of a resonant chamber analogous to a Helmholtz oscillator, consisting of a horizontally constrained fluid column on the free surface.

Experimental and theoretical observations along similar lines have been reported by Wang & Wahab (1971), for the forced vertical oscillations of two floating circular cylinders. At the closest spacing considered by Wang & Wahab (1971), the gap width between the cylinders at the free surface is equal to the cylinder radius a. In this case the added-mass force changes from a maximum positive value at $Ka = 0{\cdot}6$ to a pronounced *negative* peak at $Ka = 0{\cdot}68$, and the wave-damping force decreases from a pronounced peak at $Ka = 0{\cdot}65$ to a zero value at $Ka = 0{\cdot}8$. Wang & Wahab (1971) also consider the corresponding resonant characteristics at higher frequencies, corresponding to standing-wave modes between the cylinders and to the higher critical wavenumbers in the infinite sets of Evans & Morris (1972). Whereas there is no general relationship between the forced-oscillation problem of Wang & Wahab (1971) and the diffraction problem considered by Evans & Morris (1972) and herein, the resonant motions observed in both problems are likely to arise for the same reason, and thus this analogy is qualitatively valid.

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Note added in proof. For an analogous three-dimensional problem, an open-ended vertical pipe piercing the free surface, Isaacs & Wiegel (1949) note a similar resonant response, and find a peak magnification factor of 4·4 based on laboratory experiments with a pipe 1 ft deep and 1 in. in diameter. The present theory can be extended to analyse this problem, using the end-correction parameter for the pipe mouth computed by Levine & Schwinger (1948), and we find a corresponding magnification factor of 1000. From this comparison, and from estimates of viscous damping based on Stokes approximations for the oscillating boundary layer in the interior fluid column, we conclude that viscous losses are dominant in the three-dimensional case, but that wave radiation damping is dominant in the two-dimensional case if $b/a \gg \nu^{\frac{1}{4}} g^{-\frac{1}{8}} a^{-\frac{3}{8}}$ where $\nu =$ kinematic viscosity.

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